

Logical Nihilism: Where the Disagreement Stems from^{*}

by

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ABSTRACT: In this paper, I explore various versions of logical nihilism and situate Russell's account within the broader literature. I argue that recent objections by Dicher and Haze, as well as any analogous attempts to rebut Russell's version of logical nihilism by proposing logical systems that treat PREM and SOLO as logical constants, are ultimately unsuccessful if these proposed systems are neither universal nor equipped with any canonical application(s).

KEYWORDS: Logical Nihilism, Universalism, Pure Logic, Applied Logic, Logics of Counterexamples

ABSTRACT: In questo articolo, esploro varie versioni del nichilismo logico e colloco la teoria di Russell nel contesto più ampio della letteratura. Sostengo che le recenti obiezioni di Dicher e Haze, così come qualsiasi tentativo analogo di confutare la versione del nichilismo logico di Russell proponendo sistemi logici che trattano PREM e SOLO come costanti logiche, in definitiva falliscono se i sistemi proposti non sono né universali né dotati di applicazioni canoniche.

KEYWORDS: nichilismo logico, universalismo, logica pura, logica applicata, logiche dei controesempi

1. Introduction

Logical nihilism is a surprising position that challenges the standard views about logic. Conventionally, one either adopts logical monism (the view that only one logic is correct) or logical pluralism (the view that at least two logics are correct). However, interest in nihilism has grown, and the literature on this topic continues to expand. Logical

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nihilism requires scrutiny and clarification, because it is neither a straightforward nor a singular claim. In this paper, I aim to elucidate logical nihilism and address some of its common misinterpretations.

In her 2018 paper, *Logical nihilism: Could there be no logic?*, Gillian Russell argues that even the most widely accepted logical laws, such as reiteration, which permits the inference from P to P, have counterexamples¹. If her argument holds, it implies a form of logical nihilism, namely the view that there is no correct logic. My goal here is to examine two recent responses to Russell's argument, presented by Dicher² and Haze³, both of which rely on what I refer to as *counterexample logics*. I contend that these responses misinterpret the essence of Russell's nihilism and the function of her counterexamples.

The claim that there is no correct logic raises at least two core questions: first, what do we mean by *logic*? and second, what does it mean for a logic to be *correct*? I will show that different versions of logical nihilism make different assumptions. The version presented by Russell specifically presupposes that logic is universal, i.e., true in all domains, and that it has a special goal or canonical application. As we will see, these assumptions are overlooked in Dicher and Haze's objections to Russell.

This paper is structured as follows. Section 2 delves into various formulations of logical nihilism, and clarifies the assumptions made in each. In Section 3, I examine two recent objections to Russell, provided by Dicher and Haze, and show that these objections share a common premise. Section 4 provides my responses to these objections and illustrates why the strategy employed by Haze and Dicher fails against this account of nihilism. The concluding remarks appear in Section 5.

2. Understanding Logical Nihilism

Depending on how one interprets these terms and which terms are emphasized, various versions of nihilism emerge. In what follows, I introduce several instances of this schema, including the versions pro-

¹ G. Russell, *Logical Nihilism: Could There Be No Logic*, «Philosophical Issues» 28/1 (2018), pp. 308-324.

² B. Dicher, *Requiem for Logical Nihilism, Or: Logical Nihilism Annihilated*, «Synthese» 198/8 (2021), pp. 7073-7096.

³ T. G. Haze, *Reversing Logical Nihilism*, «Synthese» 200/3 (2022), pp. 1-18.

posed by Cotnoir⁴, Franks⁵, Russell, Dicher, Mortensen⁶, as well as certain forms I have formulated myself to complete its range of interpretations. I then situate Russell's account within this broader discussion.

We can summarize and compress various versions of logical nihilism (LN) in the following schema:

(LN) There is no correct logic/logical law⁷/consequence relation.

Russell's logical nihilism maintains that

[F]or every principle of the form $\Gamma \models \varphi$ there is an interpretation of the non-logical expressions in Γ and φ such that every member of Γ comes out true but φ does not. Such an interpretation would be a counterexample to the principle⁸.

Russell adopts the semantic (or model-theoretic) approach to logical consequence⁹, according to which the consequence relation is defined as $\Gamma \models \varphi$ iff every interpretation that makes all formula in Γ true makes φ true as well. By *logical laws*, Russell refers to principles of the form $\Gamma \models \varphi$, such as Modus Ponens ($\varphi \rightarrow \psi, \varphi \models \psi$) and Disjunctive Syllogism ($\varphi \vee \psi, \neg\varphi \models \psi$), in which the turnstile is the main predicate. While this version does not explicitly mention universalism, it presupposes it. Logic is generally regarded as universal or general, meaning that validity holds across all domains¹⁰.

To rule out the most plausible laws of FDE¹¹, namely conjunction introduction and identity, Russell uses *context-sensitivity*. One counterexample she gives is SOLO which is true when it appears alone

⁴ A. J. Cotnoir, *Logical Nihilism*, in J. Wyatt-N. Pedersen-N. Kellen (eds.), *Pluralisms in Truth and Logic*, Palgrave Macmillan, Cham-London 2018, pp. 301-329.

⁵ C. Franks, *Logical Nihilism*, in P. Rush (ed.), *The Metaphysics of Logic*, Cambridge University Press, Cambridge 2014, pp. 109-127.

⁶ C. Mortensen, *Anything Is Possible*, «Erkenntnis» 30/3 (1989), pp. 319-337.

⁷ There are various ways of defining logical laws in the literature. We may define logical laws in terms of necessity, proofs, or truth in models.

⁸ G. Russell, *art. cit.*, p. 313.

⁹ What it would mean to adopt alternative approaches to logical consequence in the context of logical nihilism remains an open question. See G. Russell, *art. cit.*, p. 312.

¹⁰ See D. Cohnitz-L. Estrada-Gonzalez, *An Introduction to the Philosophy of Logic*, Cambridge University Press, Cambridge 2019.

¹¹ FDE, or first-degree entailment is a four-valued logic containing true, false, both true and false, and neither true nor false as values. This logic has no theorems.

and false when it is embedded in a longer proposition¹²:

SOLO.
snow is white.

SOLO $\wedge$ snow is white.

For instance, if we take SOLO to be “This sentence is atomic”.

This sentence is atomic.
Snow is white.

This sentence is atomic $\wedge$ snow is white.

The above will be a counterexample of $\wedge - I$: $\varphi, \psi \models \varphi \wedge \psi$. In addition, PREM is another context-sensitive sentence that has the value T when it appears in the premises and F when it appears as a conclusion. PREM is a counterexample of the law of identity:

PREM

PREM

This is the first sentence in this argument.

This is the first sentence in this argument.

PREM and SOLO can be understood as metalinguistic variables¹³. There is a formulation of logical nihilism that closely resembles Russell’s view but explicitly emphasizes universalism (ULN). The formulation is as follows:

(ULN) There is no universal logic.

In this formulation, *correct logic* is interpreted as a logic that is correct in every domain.

Another form of nihilism, *consequence nihilism* (CLN), states that

¹² G. Russell, *art. cit.*, p. 316.

¹³ Ivi, p. 309.

(CLN) No consequence relation can have all the structural features characteristic of a logical consequence relation.

This was introduced by Dicher¹⁴. This formulation relies on structural rules¹⁵ and not logical laws. Using the jargon from sequent calculus, this means that all the structural rules have counterexamples.

To better understand the claim that there is no correct logic, we must first clarify what it means for a logic to be correct. In the formulation above, correctness is understood to include the structural features that a logical consequence relation must exhibit¹⁶.

Another plausible way to account for the correctness of a logic is to identify a canonical application for it: a special job that we want our logic to do. These special jobs must be able to induce rivalry among logics: that is, it must be possible to ask which of the available logical systems do the job better. Of course, not every application will be canonical on this definition. There can be different possible applications for non-pure logics based on what aim logicians have in mind. For Russell, following in a long philosophical tradition, the canonical application is universal in scope: the correct logic must characterize correct reasoning in every domain of inquiry. But for others, such as Stewart Shapiro, the canonical application of logic is capturing mathematical reasoning.

A version of nihilism that includes a canonical application (CALN) is offered by Cotnoir¹⁷. This formulation is stronger than the previous version:

(CALN) There is no logical consequence relation that correctly represents natural language inference.

The essential feature of this version is that the presupposed job of logic is to represent natural language consequence. Therefore,

¹⁴ See B. Dicher, *art. cit.*, p. 7082.

¹⁵ Structural rules are inference rules that do not involve logical connectives. Examples include contraction, cut, weakening, and exchange.

¹⁶ Correctness can depend on multiple conditions, including a set of structural features as one relevant factor. Although standard natural deduction systems for both classical and intuitionistic logic share the same structural rules, some may still claim that intuitionistic logic is correct while classical logic is not. I thank an anonymous reviewer for drawing my attention to this point.

¹⁷ See A. J. Cotnoir, *art. cit.*, p. 2.

it is easier to decide whether Cotnoir's logical nihilism is correct. In general, Cotnoir's version would be easier to deal with, as it has a clear stand on what logicity is and what application logic has. Nevertheless, Russell's and Dicher's versions are independent of particular applications of logic and for them, the job of logic is open to interpretations. As we will see, Russell's account maintains that a logic must have a special job, for example, be useful for mathematics or can play the role of the metalogic. Moreover, according to Dicher, logic does not have to follow a specific job. Even so, I believe we cannot dismiss any form of logical nihilism by claiming that logic has no special job/canonical application. This is partly how Dicher attempts to refute Russell's nihilism.

We can generalize Cotnoir's nihilism (G-CALN) by taking other possible canonical applications into account:

(G-CALN) For each canonical application, there is no logical consequence relation that correctly represents it.

This formulation is a bit different from what Russell has suggested¹⁸. If we distinguish canonical applications and consider each separately, we will get this version of nihilism. This formulation is stronger than the next, weaker formulation (W-CALN):

(W-CALN) For some canonical applications, there's no logical consequence relation that correctly represents it.

This form may be correct if only the logic for the supposed canonical applications ends up empty. Cotnoir's logical nihilism is a special case of this form because it considers the canonical application of logic fixed and unique¹⁹.

¹⁸ Unlike the version discussed above, Russell's account of logical nihilism appeals to the generality of logical laws, as it is demonstrated by the following argument: «To be a law of logic, a principle must hold in complete generality. No principles hold in complete generality. Therefore, there are no laws of logic» (G. Russell, *art. cit.*, p. 308).

¹⁹ While the correctness of logics is a requirement for formulating logical nihilism, Franks believes the question of truth is an irrelevant question to ask (C. Franks, *Logical Nihilism*, in Å. Hirvonen-J. Kontinen-R. Kossak-A. Villaveces (eds.), *Logic Without Borders: Essays on Set Theory, Model Theory, Philosophical Logic and Philosophy of Mathematics*, De Gruyter, Berlin-München-Boston 2015, pp. 147-166).

Finally, Mortensen²⁰ has suggested the following version (MLN):

(MLN) Everything is a logical possibility.

This account assumes that for every argument, there is a case in which the premises are true, but the conclusion is false. This formulation depends on inferences and not logical laws or rules. Therefore, whereas we need a whole logic to be a counterexample for other versions of logical nihilism, one valid inference is sufficient to reject Mortensen's logical nihilism.

As we have seen, what most of these formulations of logical nihilism have in common is denying the correctness of either logic/ logical laws/consequence relation. But the criteria for *correctness* can vary from one to the other. Whereas in some formulation universalism matters and is a part of correctness, in the rest it does not. Furthermore, the application/purpose of logic is essential in some of the formulations. Nonetheless, it is not as important in other formulations. Hence, in some of the versions of logical nihilism, correctness depends on the purpose/application. In the next section, I will introduce one of the suggested solutions to Russellian logical nihilism.

3. Counterexamples Logics

The objection maintains that Russell's logical nihilism is self-refuting. For the view to be self-refuting, it must imply at least one correct logical law, or worse yet, a complete logic. Henceforth, I will refer

According to him, the question of the correctness of logics has been overrated, and this will halt progress in logic because sticking to a logic, or a few logics will prevent us from thinking about new logic(s) and new applications. Although I agree with Franks that it is important to be able to account for different applications of logic, and a single or a limited number of logics cannot accommodate this, I believe this does not show that it is wrong to inquire after (a) true logic(s). In addition, the general question of disparate applications of logic is something that even prominent monists do not mind calling themselves pluralists about. What matters here is what kind of application we are talking about, canonical, or non-canonical. We must distinguish between applications when we are thinking about the truth of logical laws. In the case of canonical applications, searching for correct logic(s) is a meaningful endeavor.

²⁰ See C. Mortensen, *art. cit.*

to the logics Dicher²¹ and Haze²² propose, as *counterexample logics*, which arise by treating the counterexamples as logical constants.

3.1 Dicher's Counterexample Logic

Dicher argues that the counterexamples PREM and SOLO do not have the form Russell claims. By introducing new logical constants to FDE²³, we can ostensibly accommodate these putative counterexamples. Of course, a question arises as to whether it is appropriate to treat PREM and SOLO as logical constants²⁴. For the purposes of this paper, I will assume that PREM and SOLO can indeed be treated as logical constants and proceed to introduce the counterexample logic.

The logic proposed by Dicher is an extension of FDE that includes validities such as: $\emptyset \vdash \neg \text{PREM}$; $\emptyset \vdash \neg \text{PREM} \vee \text{PREM}$; $\text{PREM} \vdash \neg \text{PREM}$; $p \vdash \neg \text{PREM}$; $p \vee q \vdash \neg \text{PREM}$; $\emptyset \vdash \text{SOLO}$; $\emptyset \vdash \neg \text{SOLO}$; $p \vdash \neg \text{SOLO} \vee q$. The validities of this logic are all generated using logical constants. Although these validities are not logical laws, they are instances of such laws. The resulting logic is substitution invariant, meaning that for every substitution σ , if $X \vdash A$, then $\sigma(X) \vdash \sigma(A)$. Dicher uses the term logical minimalism to describe the form of nihilism obtained when treating PREM and SOLO as logical constants. According to this view, there are no correct logical laws, yet there are correct²⁵ logics, such as the one Dicher proposes.

To strengthen his counterargument against Russell's and conse-

²¹ See B. Dicher, *art. cit.*

²² See T. G. Haze, *art. cit.*

²³ Logical constants can be defined either proof-theoretically or semantically. In a proof-theoretic definition, we focus on the form of inference, with logical constants shaping that form. Under the model-theoretic account, logical constants determine the truth conditions of inferences.

²⁴ Russell (*art. cit.*) maintains that not every arbitrary term can be admitted as a logical constant, whereas Dicher (*art. cit.*) contends that nothing prevents us from accepting PREM and SOLO as such.

²⁵ Here, it is not specified what correctness refers to. The nihilist's view of correctness contrasts with that of Dicher. Dicher conceives of logic as a cognitive tool; accordingly, correctness should be defined within that framework. On this view, logic «not only models (aspects of) the pre-existing practice, but it also fashions extensions, improvements, etc. of it. Standards of deductive validity, for instance, are not given alongside natural language. They are developed by logicians and used, among other things, to extend the scope, precision, reliability of our deductions» (B. Dicher, *art. cit.*, p. 7092).

quence nihilism, and rebut logical minimalism, Dicher introduces a meta-inferential account of logical laws within sequent calculus²⁶. He proposes taking meta-inferences as logical laws instead of sequents (inferences)²⁷. This will result in a logic without non-trivial valid inferences but still equipped with logical laws that are meta-inferences. A meta-inference is a sequent-to-sequent inference in a sequent calculus. Thus, rather than recognizing just one-line sequents as laws, a meta-inference connects one line to another line. For example,

$$\frac{p \vdash p \quad q \vdash q}{p, p \supset q \vdash q}$$

represents a meta-inference, which is the $L \supset$ rule. Sequent calculus derivations are structured as proof trees, with the desired sequent at the root and axioms, typically instances of reflexivity, at the leaves. Therefore, instead of continuing to treat inferences like “if A then A” as laws, Dicher suggests accepting meta-inferences (such as the one above) as the genuine logical laws.

Dicher adopts a consequence relation that retains key Tarskian features – namely, reflexivity, monotonicity, and transitivity. However, unlike the traditional Tarskian consequence relation, which operates at the level of formulas, this alternative operates at the level of sequents. The traditional Tarskian consequence relation is defined as follows:

(*Tarskian consequence*) Let $\mathcal{L}$ be a propositional language, $Form(\mathcal{L})$ the set of its formulae, and $\mathcal{P}(Form(\mathcal{L}))$ its power set. A logical consequence relation over $\mathcal{L}$ is a substitution-invariant relation $\vdash \subseteq \mathcal{P}(Form(\mathcal{L})) \times Form(\mathcal{L})$ satisfying, for all $X, Y \subseteq \mathcal{L}$ and all $A, B \in Form(\mathcal{L})$:

- (1) if $A \in X$, then $X \vdash A$ (Reflexivity);
- (2) if $X \vdash A$ and $X \subseteq Y$, then $Y \vdash A$ (Monotonicity); and
- (3) if $X \vdash A$ and, for every $B \in X$, $Y \vdash B$, then $Y \vdash A$ (Transitivity)²⁸.

One result of the Tarskian framework is that it excludes substructural logics.

²⁶ For a detailed overview of sequent calculus, see S. Negri-J. von Plato-A. Ranta, *Structural Proof Theory*, Cambridge University Press, New York 2001.

²⁷ A sequent is an inference of the form $\Gamma \vdash \Delta$, where Γ and Δ are finite sets.

²⁸ B. Dicher, *art. cit.*, p. 7083.

Dicher uses the following definition of validity:

(*Local meta-inferential validity*) Let $S_1, \dots, S_m, S_n$ be inferences (sequents). A meta-inference with premises $S_1, \dots, S_m$ and conclusion S_n is *locally valid* iff, for every valuation v , either v does not satisfy S_i , for some $i \in \{1, \dots, m\}$, or v satisfies S_n ²⁹.

This definition is a generalization of the definition of validity for inferences. According to this definition, even if reflexivity cannot be achieved at the level of formulas, it can still hold for sequents. This consequence relation is known as the *Blok-Jonsson consequence relation*, which is defined over arbitrary sets and not solely over sets of formulas.

For instance, the meta-inference

$$\frac{PREM \vdash SOLO}{SOLO \vdash SOLO}$$

is locally valid. This is because every valuation v , satisfies $SOLO \vdash SOLO$. On the other hand, the meta-inference

$$\frac{p \wedge PREM \vdash q \wedge p}{p \vdash q \wedge PREM}$$

is not locally valid. A valuation v , such that $v(p) = v(q) = 1$, will satisfy the premise inference, however it does not satisfy the conclusion inference.

By shifting to the meta-inferential level, Dicher aims to preserve logical laws by generalizing their formulation from an inferential level to a meta-inferential one. However, it remains unclear whether this move prevents other kinds of counterexamples. As we will see in section 4.2, we can still generalize logical nihilism to this higher-order consequence relation.

3.2 Haze's Counterexample Logic

Analogous to Dicher, Haze proposes a counterexample logic by using

²⁹ Ivi, p. 7085.

sentences like SOLO and PREM, which he refers to as SOLO-Only sentences, formatting them in italics. Haze constructs a logic that he calls SOLO-Only Propositional Logic. SOLO-Only sentences are sentences that act like SOLO. This logic contains inferences such as

Grass is green $\rightarrow$ *snow is white*.
Therefore, $\neg$ grass is green³⁰,

where *snow is white* is a SOLO-Only sentence. In line with Dicher, Haze's intention for constructing this logic is to show that even though the counterexamples are fallacious, there can be still valid arguments containing SOLO and PREM. And in fact, these instances of valid arguments can be numerous. Here is the system proposed by Haze:

Solo-Only Propositional Logic (SOPL).

Vocabulary:

Propositional letters:

Normal: $a, b, c, \dots, a_i, b_i, c_i, \dots$

Solo-only: $a, b, c, \dots, a_i, b_i, c_i, \dots$

Connectives: $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$.

Brackets: ()

Syntax:

– Propositional letters (both normal and solo-only) are wffs.

– If A and B are wffs, then so are:

$\neg A$

$(A \wedge B)$

$(A \vee B)$

$(A \rightarrow B)$

$(A \leftrightarrow B)$

– Nothing else is a wff³¹.

The semantics of this logic consist of two parts: the first part involves sentences that are not of the form SOLO (the normal part), while the second part includes inferences involving SOLO-like sentences.

For any two normal wffs A and B :

$\neg A$ is true iff A is false.

$(A \wedge B)$ is true iff A and B are both true.

$(A \vee B)$ is true iff A is true or B is true (or both).

³⁰ See T. G. Haze, *art. cit.*, p. 14.

³¹ T. G. Haze, *art. cit.*, p. 13.

$(A \rightarrow B)$ is true iff it is not the case that A is true and B is false.

$(A \leftrightarrow B)$ is true iff A and B are both true or both false.

Where A is a normal wff and a and b are SOLO-Only wffs:

$\neg a$ is true.

$(A \wedge a)$ is false.

$(a \wedge A)$ is false.

$(a \wedge b)$ is false.

$(A \vee a)$ is true iff A is true.

$(a \vee A)$ is true iff A is true.

$(a \vee b)$ is false.

$(A \rightarrow a)$ is true iff A is false.

$(a \rightarrow A)$ is true.

$(a \rightarrow b)$ is true.

$(A \leftrightarrow a)$ is true iff A is false.

$(a \leftrightarrow A)$ is true iff A is false.

$(a \leftrightarrow b)$ is true.

[...]

Definition of consequence and tautology:

For any set of wffs of Γ and any wff α , $\Gamma \models \alpha$ [...] iff there is no model M on which all members of Γ are true and α is false.

α is a tautology iff $\emptyset \models \alpha$ ³².

Haze gives the following examples of valid forms³³:

$a \vee A$.

Therefore, A .

Since a is a SOLO-Only sentence, it is false when it is embedded in $a \vee A$. As a result, the truth value of $a \vee A$ is the same as the truth value of A . Hence, there is no valuation that makes the premise true and the conclusion false.

4. *The Rebuttal*

In this section, I argue that the strategy of deriving new logics from the counterexamples *PREM* and *SOLO* does not succeed. There are two main reasons for this. First, these counterexample logics do not

³² Ivi, pp. 14-15.

³³ Ivi, p. 15.

have canonical applications, whereas in Russellian logical nihilism, logic is supposed to possess such canonical applications. Second, these counterexample logics are not universal, yet logic is assumed to be universal in this version of logical nihilism. Finally, if one takes meta-inferences to be logical laws in line with Dicher³⁴, a new form of logical nihilism emerges.

4.1 Logics of Counterexample Are Fruitless and Not Universal

One way to assess the correctness of a logic is by identifying its canonical applications, in other words, the special tasks we expect our logic to perform. These tasks must be capable of motivating rivalry among different logics: they allow us to ask which of the available systems does the job more effectively. Of course, not every application will be canonical according to this definition. There may be various intended goals for non-pure logics, depending on the aims of those developing them.

For Russell, as already noted (section 2), the canonical application is universal in scope: the correct logic must characterize sound reasoning across every domain of inquiry³⁵. By contrast, other philosophers, for instance Stewart Shapiro, take mathematical reasoning to be the canonical application of logic. Commandeur describes these divergent applications as follows:

One prominent candidate for the primary goal of logic is that of a formal codification of logical consequence in natural language [...]. Cook³⁶ [...] has proposed this. A different goal, proposed by McSweeney³⁷ [...], is that of capturing the structure of mind-and-language independent reality. Yet another goal, though not explicitly defended as the sole primary goal of logic, is to model information-flow in a dynamic, multi-agent setting, as the logical dynamics program has it³⁸.

³⁴ B. Dicher, *art. cit.*

³⁵ G. Russell, *art. cit.*

³⁶ R. T. Cook, *Let a Thousand Flowers Bloom: A Tour of Logical Pluralism*, «Philosophy Compass» 5/6 (2010), pp. 492-504.

³⁷ M. M. McSweeney, *Logical Realism and the Metaphysics of Logic*, «Philosophy Compass» 14/1 (2019), pp. 1-10.

³⁸ L. Commandeur, *Against Telic Monism in Logic*, «Synthese» 200/1 (2022), pp. 1-18, p. 2.

We should consider the notion of a canonical application in the context of applied logic. The distinction between applied and pure logic parallels that between applied and pure geometry. In pure geometry, which need not address how the universe is actually structured, one studies geometry primarily for its own sake. This is similar to much of mathematical logic, which investigates formal features in a purely theoretical manner, with no external purpose in mind. By contrast, philosophical logic can be likened to applied geometry, in the sense that both aim to capture something external. Applied geometry strives to represent either the geometry of the universe or that of some specific domain; likewise, philosophical logic addresses topics of significant philosophical interest and, accordingly, belongs on the “applied” side³⁹. This analogy underscores the importance of having a specific purpose in philosophical logic. Without such a purpose, the boundary between pure and applied logic would vanish, and we would effectively lose sight of what makes philosophical logic distinct. Priest expresses this as follows:

There are many *pure* geometries [...]. Rivalry between them can arise only when they are applied in some way. Then we may dispute which is the correct geometry for a particular application, such as mensurating the surface of the earth [...]. Geometry had what one might call a canonical application: the spatiotemporal structure of the physical cosmos. Indeed this application was coeval with the rise of Euclidean geometry [...]. But exactly the same picture holds with respect to logic. There are many pure logics: classical logic, intuitionist logic, various paraconsistent logics, and so on. And as pieces of pure mathematics, all are equally good. They all have systems of proof, model theories, algebraicisations. Each is a perfectly good mathematical structure. But pure logics are applied for many purposes: to simplify electrical circuits (classical propositional logic), to parse grammatical structures (the Lambek calculus), and it is only when different logics are taken to be applied for a particular domain that the question of which is right arises. Just as with geometries, each applied logic provides, in effect, a theory about how the domain of application behaves⁴⁰.

³⁹ D. Cohnitz-L. Estrada-Gonzalez, *op. cit.*

⁴⁰ G. Priest, *Revising Logic*, in P. Rush, *The Metaphysics of Logic*, cit., pp. 211-223, p. 215.

In response to the counterexample logics presented by Dicher⁴¹ and Haze⁴², Russell would likely offer the same reply she gives to the objection that the consequence relation remains non-empty due to trivial validity instances such as an inference with $\top$ as its premise and $\top \vee \top$ as a conclusion, where $\vee$ is a 0-place truth-functor and is always interpreted as true⁴³. She would argue that this sort of logic is useless, as it neither serves as a metalogic nor benefits mathematics⁴⁴. While that may be correct, the possibility remains that logic might have an application outside these two domains; it need not be restricted to metalogic or mathematics alone. In response to this point, Dicher argues that «there can be no privileged position from which to assess the usefulness of logic»⁴⁵. This stance allows critics of logical nihilism to reject the nihilists' final step: ruling out the adequacy of minimalism. However, the price for this move is reducing *logic* to pure logic, entirely divorced from any genuine philosophical purpose. Logic without a canonical application cannot be judged for correctness.

The fact that we lack a method for selecting between possible canonical applications does not imply that philosophical logic has no essential purpose(s) or canonical application(s). Indeed, the cost of accepting a domain-specific logical pluralism is lower than adopting Dicher's thesis that equates philosophical logic with pure logic by denying that logic can have canonical applications⁴⁶. According to domain-specific logical pluralism (DLP), different logical systems are suitable for different domains or contexts. At a minimum, logical pluralism more closely reflects the practice of philosophical logicians in capturing the aims and applications they pursue. By contrast, claiming that philosophical logic is identical to pure logic is far more radical. Mathematical and pure logicians do not do the same work as philosophical logicians, which is why we should not disconnect philosophical logic from its applications. After all, that is how philosophical logic is defined.

From a Peircean standpoint, the distinction between applied logic

⁴¹ B. Dicher, *art. cit.*

⁴² T. G. Haze, *art. cit.*

⁴³ Such validities hold only vacuously, because either the left side or the right side of the inference always holds. These are not logical laws; they are only instances of logical laws.

⁴⁴ G. Russell, *art. cit.*, p. 322.

⁴⁵ B. Dicher, *art. cit.*, p. 7080.

⁴⁶ B. Dicher, *art. cit.*

(*logica utens*) and pure logic (*logica docens*) concerns extra-systematic versus system-relative validity⁴⁷. One way to confer purpose on logic is to task it with capturing our intuitions about the validity or invalidity of informal arguments⁴⁸. In contrast, *logica docens* involves the study of formal arguments and makes no claim about extra-systematic validity, i.e., whether a given informal argument is ultimately valid. On the other hand, *logica utens* explicitly addresses such informal arguments and extra-systematic validity⁴⁹. Dismissing logical nihilism simply because one does not believe in *logica utens* is certainly an option, but anyone taking that position should clarify where they stand on this issue and, if necessary, provide an argument for rejecting extra-systematic validity⁵⁰.

A similar objection applies to Haze's account⁵¹. Even if SOPL is a logical system, what application does it serve? The most obvious function is to examine valid arguments within contexts involving SOLO-like sentences. However, this purpose is not a canonical one; at best, it is only part of a canonical application, as it focuses specifically on the behavior of SOLO. This might have been compelling if SOLO and PREM had been chosen as logical constants in a more systematic way. Instead, their designation as logical constants appears rather *ad hoc*: they were introduced specifically to counter logical nihilism. Consequently, it seems implausible that a logic featuring SOLO or PREM as logical constants would capture actual fragments of natural language. Therefore, even if these counterexample logics were universal, without a separate argument demonstrating that the counterexamples truly function as logical constants, they do not successfully refute Russellian logical nihilism as self-defeating.

Another reason that Dicher⁵² and Haze⁵³ appear to misunder-

⁴⁷ Priest makes a similar distinction. For him, *logica utens* refers to the logic we employ in our reasoning, *logica docens* to our theoretical account of logic, and *logica ens* to extra-systematic validity. I thank an anonymous reviewer for bringing this to my attention. See G. Priest, *art. cit.*

⁴⁸ This is only one example of a possible canonical application for logic.

⁴⁹ S. Haack, *Philosophy of Logics*, Cambridge University Press, Cambridge 1978, pp. 14-16.

⁵⁰ As already noted, Dicher, for example, maintains this position by claiming that «there can be no privileged position from which to assess the usefulness of logic» (B. Dicher, *art. cit.*, p. 7080).

⁵¹ T. G. Haze, *art. cit.*

⁵² B. Dicher, *art. cit.*

⁵³ T. G. Haze, *art. cit.*

stand Russell's account⁵⁴ is that their counterexample logics lack universality. Dicher contends that we must reject universalism:

Needless to say, there being (putative) counterexamples to logically valid inferences does not mean that there are no cases in which these inferences hold. It just means that they do not hold in every case and so do not hold logically. Ultimately logic gets into trouble because it is (usually taken to be) general: there are no exceptions to its laws, which apply across every domain of inquiry, irrespective of the particular features of that domain. However, this kind of generality is fragile. If there are no legitimate constraints that could be imposed to safeguard it, it is untenable⁵⁵.

Here is precisely where the misunderstandings arise. Russell herself attempts to block nihilism through lemma incorporation⁵⁶, aiming to eliminate the counterexamples by modifying logical laws⁵⁷.

4.2 Meta-Inferential Logical Nihilism

My second criticism of Dicher's approach is that moving to the meta-level to escape logical nihilism does not ultimately resolve the core issue. Broadening the syntax in new ways reveals that treating meta-inferences as logical laws can introduce additional problems. First, transitivity as a meta-inference may give rise to counterexamples. If we accept transitivity, we might be unable to capture transparent truth, which is often added to a logic via the following rules:

$$\frac{\Gamma, A \vdash \Delta}{\Gamma, T \langle A \rangle \vdash \Delta} \text{LT}$$

⁵⁴ G. Russell, *art. cit.*

⁵⁵ B. Dicher, *art. cit.* p. 7074.

⁵⁶ See G. Russell, *art. cit.*

⁵⁷ It should be noted that lemma incorporation is not a guaranteed solution for fixing logical laws. As Wyatt and Payette points out, a background logic is needed for these modifications to hold, yet it remains unclear which logic should play that role. See N. Wyatt-G. Payette, *Against Logical Generalism*, «Synthese» 198/20 (2021), pp. 4813-4830.

$$\frac{\Gamma \vdash A, \Delta}{\Gamma \vdash T < A >, \Delta} \text{RT}$$

Transparent truth and transparent validity lead to some well-known paradoxes like the liar paradox and the Curry paradox. Assuming that κ is $T < \kappa > \rightarrow p$ for some absurd p , the Curry paradox follows:

$$\frac{\frac{\frac{T < \kappa > \vdash T < \kappa > \quad p \vdash p}{T < \kappa >, T < \kappa > \rightarrow p \vdash p} LC}{\frac{T < \kappa >, \kappa \vdash p}{T < \kappa > \vdash p} RT, Contr.} Def. \quad \frac{\frac{\frac{T < \kappa > \vdash T < \kappa > \quad p \vdash p}{T < \kappa >, T < \kappa > \rightarrow p \vdash p} LC}{\frac{T < \kappa >, \kappa \vdash p}{T < \kappa > \vdash p} RT, Contr.} Def.$$

$$\frac{\frac{\frac{\vdash \kappa}{\vdash T < \kappa >} RT}{\vdash p} \quad \frac{\frac{\frac{T < \kappa > \vdash T < \kappa > \quad p \vdash p}{T < \kappa >, T < \kappa > \rightarrow p \vdash p} LC}{\frac{T < \kappa >, \kappa \vdash p}{T < \kappa > \vdash p} RT, Contr.} Def.$$

$$\vdash p \quad Cut$$

The liar sentence λ states that “ λ is not true”. So, here a self-referential sentence will lead to a genuine paradox. Now we can derive the liar paradox:

$$\frac{\frac{\frac{\lambda \vdash \lambda}{T < \lambda > \vdash \lambda} LT}{\vdash \neg T < \lambda >, \lambda} RN}{\vdash \lambda} Def, Contr. \quad \frac{\frac{\frac{\lambda \vdash \lambda}{\lambda \vdash T < \lambda >} RT}{\neg T < \lambda >, \lambda \vdash} LN}{\lambda \vdash} Def, Contr.$$

$$\vdash \quad Cut$$

Transitivity is commonly identified as the source of these paradoxes. Responses to these paradoxes vary: some logicians, who aim to accommodate transparent truth in the object language, reject the Cut rule⁵⁸. So,

⁵⁸ See, for instance, D. Ripley, *Paradoxes and Failures of Cut*, «Australasian Journal of Philosophy» 91/1 (2013), pp. 139-164; E. Barrio-L. Rosenblatt-D. Tajer, *Capturing Naive Validity in the Cut-Free Approach*, «Synthese» 199/3 (2016), pp. 707-723; P. Cobreros-P. Egge-D. Ripley-R. van Rooij, *Reaching Transparent Truth*, «Mind» 122/488 (2013), pp. 841-866.

$$\frac{\vdash T < \kappa > \quad T < \kappa > \vdash p}{\vdash p} \text{ Cut}$$

can be seen as a counterexample to the rule Cut. Note that Cut is a meta-inference.

Even meta-inferential reflexivity proves vulnerable.

$$\frac{\vdash \text{Con-antecedent}}{\vdash \text{Con-antecedent}} \text{ Sref}$$

Let *Con-antecedent* be a sentence that is declared true when it appears as the conclusion of the antecedent sequent and false when it appears as the conclusion of the succedent inference. This meta-inference is not locally valid and thus provides a counterexample to reflexivity as a meta-inferential logical law.

Advocates of Tarskian consequence may choose to relinquish the ability to represent transparent truth. Others typically reject transitivity or contraction. The key point is that, by extending our syntax to accommodate a truth predicate within our language, significant problems arise, particularly involving transitivity⁵⁹.

⁵⁹ There is yet another logic of counterexample worth considering, different from the two previously mentioned in that it is actually useful and serves a genuine purpose (i.e., it has a canonical application). See A. Fjellstad, *Logical Nihilism and the Logic of "Prem"*, «Logic and Logical Philosophy» 30/1 (2020), pp. 311-325. Fjellstad recognizes that "usefulness" is part of Russell's logical nihilism: «We can easily tweak the interpretation of familiar logical constants such as $\neg$ and $\rightarrow$ to thereby obtain non-reflexive logics for prem with what we can describe as useful valid inferences that support the uniform substitution of any formula for propositional variables» (ivi, p. 313). Non-reflexive logics can serve the purpose of blocking set-theoretic and proof-theoretic paradoxes, a goal potentially significant enough to qualify as a canonical application. As Fjellstad notes, his proposal is not intended as a counterexample to Russell's logical nihilism. Instead, the non-reflexive logic he develops is not meant to be universal or valid in all domains, but rather to provide a way around these paradoxes. Whereas other non-reflexive logics in the literature generally fail to preserve uniform substitution, Fjellstad introduces a non-reflexive logic that treats PREM as a logical constant by adjusting the interpretation of negation so that modus tollens still holds, and modifying the interpretation of the conditional, so that modus ponens likewise remains valid. Naturally, the earlier objections to treating PREM or SOLO as logical constants also apply here.

5. Conclusion

Both Dicher⁶⁰ and Haze⁶¹ regard logical nihilism as self-refuting, given that they can still produce their respective counterexample logics. However, as I have argued, Dicher's defense of logical minimalism, namely, that minimalism does not amount to nihilism, is not convincing⁶². These logics fail to address many versions of logical nihilism because they do not specify what it means for a logic to be correct. Even if we acknowledge that they are legitimate logics, they capture phenomena insufficiently significant to qualify as correct logics in the sense required by logical nihilism, whose core claim concerns the criteria of correctness that any genuine logic must satisfy. While this does not refute Russell's brand of nihilism, it does undermine Mortensen's version⁶³, which posits that there are no correct arguments at all. In contrast, as long as one valid argument exists, that position collapses. These purported counterexample logics demonstrate that many arguments are free of counterexamples.

In clarifying logical nihilism, it is crucial to explain precisely what is meant by *logic* and *correctness*. A commitment to universality and a specific standard of correctness shape the variety of logical nihilism in question. Consequently, more than one thesis may rightfully be called *logical nihilism*, and anyone arguing for or against a particular version must specify which version to avoid equivocation. As Russell emphasizes, both universality and canonical application matter when dealing with *philosophical logic*⁶⁴.

I have shown that neither Dicher's nor Haze's responses to Russell succeed. The logics they propose lack any canonical application, and redefining validity to mean *local validity* merely introduces fresh counterexamples (such as the Con-antecedent example). Finally, replacing

⁶⁰ B. Dicher, *art. cit.*

⁶¹ T. G. Haze, *art. cit.*

⁶² One might say the same about Haze's contention that arguments involving SOLO and PREM commit the fallacy of equivocation (see T. G. Haze, *art. cit.*). Critiquing that portion of Haze's argument lies beyond the scope of this paper.

⁶³ C. Mortensen, *art. cit.*

⁶⁴ G. Russell, *art. cit.*

inferences with meta-inferences is equally unhelpful, for although it may sidestep complications tied to SOLO and PREM, the principle of transitivity still spawns paradoxes.

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